MULTI-OBJECTIVE OPTIMIZATION OF MULTIPLE DEPOT VEHICLE ROUTING PROBLEM USING SWARM INTELLIGENCE TECHNIQUES

A Synopsis submitted in the partial fulfilment of the requirement for the award of the degree of

Doctor of Philosophy
in
Computer Science

Submitted by
Rohit Sharma

Under the Supervision of

Dr. Sanjay Saini

G.S. Tyagi
Professor,
Head & Dean,
Dept. of Physics & Computer Sc.
Faculty of Science

DEPARTMENT OF PHYSICS AND COMPUTER SCIENCE
FACULTY OF SCIENCE
DAYALBAGH EDUCATIONAL INSTITUTE
DAYALBAGH, AGRA (UP) – 282005
JANUARY 2018

1. Introduction

Physical distribution is one of the key functions in supplying systems, involving the flow of product from manufacturing plants or distribution centers through the transportation network to clients. It is a very costly function, especially for the distribution industries. The operational research literature has addressed this problem by calling it as the Vehicle Routing Problem (VRP). The VRP refers to one of the combinatorial optimization problems in which clients are to be served by a number of vehicles. The vehicles leave the depot, serve clients in the network and return to the depot after completion of their routes. Each client is described by a certain demand.

From the complexity point of view, the classical VRP is understood to be NP-hard since it generalizes TSP (travelling salesman problem) and BPP (bin packing problem) which are both well known NP-hard problems. The problem consists of determining a set of vehicle routes in such a way that: (i) each client is serviced exactly once by a vehicle, (ii) each vehicle route starts and ends at the same depot, (iii) the total demand of each route does not exceed the vehicle capacity, and (iv) the total cost of the distribution is minimized.

The problems concerning the distribution of goods between depots and clients are generally known as VRPs (Vehicle Routing Problems) or Vehicle Scheduling Problems. Typical applications of VRPs are, solid waste collection, school bus routing, transportation of handicapped persons, street cleaning, dial-a-ride systems, routing of salespeople and of maintenance units.

The VRP is responsible in designing a best set of route for a fleet of vehicles in order to serve a given set of clients. The interest in Vehicle Routing Problem is motivated by its practical relevance. During the past few decades an increasing number of optimization techniques based on operations research are proposed, for the effective management of the provision of the goods and services in distribution systems. The VRP have several variants based on the operational mechanism and mathematical modelling. Based on the constraints and characteristics of VRP, they are classified into periodic VRP, split delivery VRP, multi-depot VRP, capacitated VRP, stochastic VRP, VRP with time window, VRP with pick-up and delivery and VRP with backhauls etc.

The Multi-depot VRP (MDVRP), an extension of classical vehicle routing problem, is an NP-hard problem for simultaneously finding the routes for various vehicles from multiple depots to a set of clients and then returning to the same depot. The goal of the problem is to determine routes for vehicles such that all the clients are served at a minimum cost in terms of total travel distance, without disturbing the capacity and travel time constraints of the vehicle.

According to MDVRP, the number and locations of the depots are predetermined. Each depot is large enough to store all the products ordered by the clients. Each vehicle starts and finishes at the same depot. The demand and location of each client is also known in advance and each client is visited by a vehicle exactly once. Figure 1 shows an example of the Multi-Depot VRP with two depots and ten clients. Since there are additional depots for storing the products, the decision makers have to find depots through which the clients are served. The decision making steps are described into grouping, routing, scheduling and optimization as shown in Figure 2.

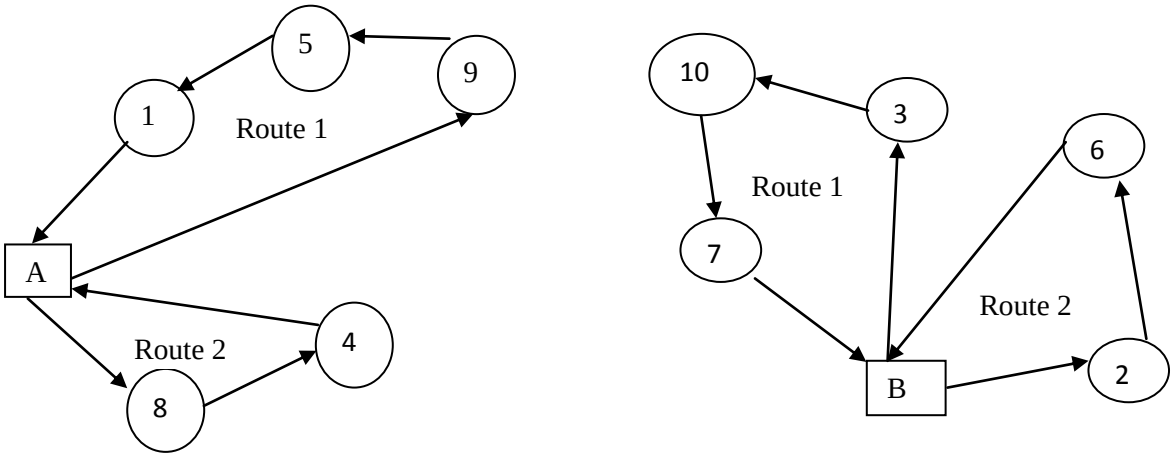

Figure 1 Example of an MDVRP with 2 depots & 10 customers

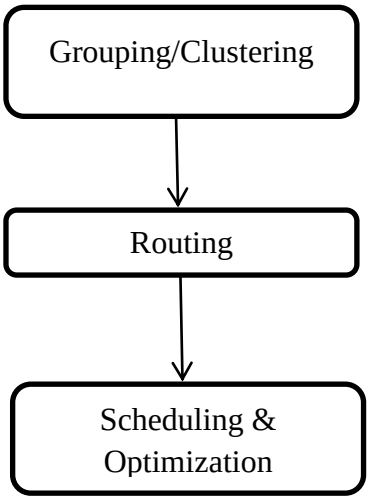

Figure 2 Decision making in MDVRP

In grouping, clients are clustered based on distance among clients and depots. In the example, clients 1,5,9,4,8 are assigned to depot A while clients 7,10,3,6,2 are assigned to depot B. In depot A, clients 1,5,9 are within the first route, while clients 4 and 8 are served in the second route. The clients of the same depot are assigned to several routes in the routing phase by means of Clarke and Wright saving method and each route is sequenced in the scheduling phase. The aim of routing is to reduce the number of routes without disturbing the capacity constraints. Since there are 2 depots, the minimum number of routes can be limited to 2. More number of routes increase the number of vehicles required as a result decreasing the distance. In general, the goal of the MDVRP is to reduce the total delivery distance or time spent in serving all clients thus utilizing efficient amount of vehicles.

2. Problem Description

The VRP is one of the most challenging combinatorial optimization tasks in real time logistics applications. The VRP may be defined as a problem of finding the optimal routes of shipping or collection from one or several depots to some of cities or clients, while satisfying capacity and time constraints. In a real-world environment, drivers pick out the shortest path to reach a destination, because of this, the distance travelled and the cost can be minimized. Collection of household garbage, fuel shipping trucks, goods distribution, and snow plough, road cleansing, dial-a-ride systems, school

bus routing transportation, routing of salespeople and mail shipping are the most common applications of the VRP. The VRP plays a critical role in distribution, logistics and supply chain management. Huge studies efforts have been devoted to studying the VRP since 1959 where Dantzig and Ramser (G. B. Dantzig, , & J. H. Ramser, 1959) have defined the trouble as a generalized problem of TSP (Travelling Salesman Problem).

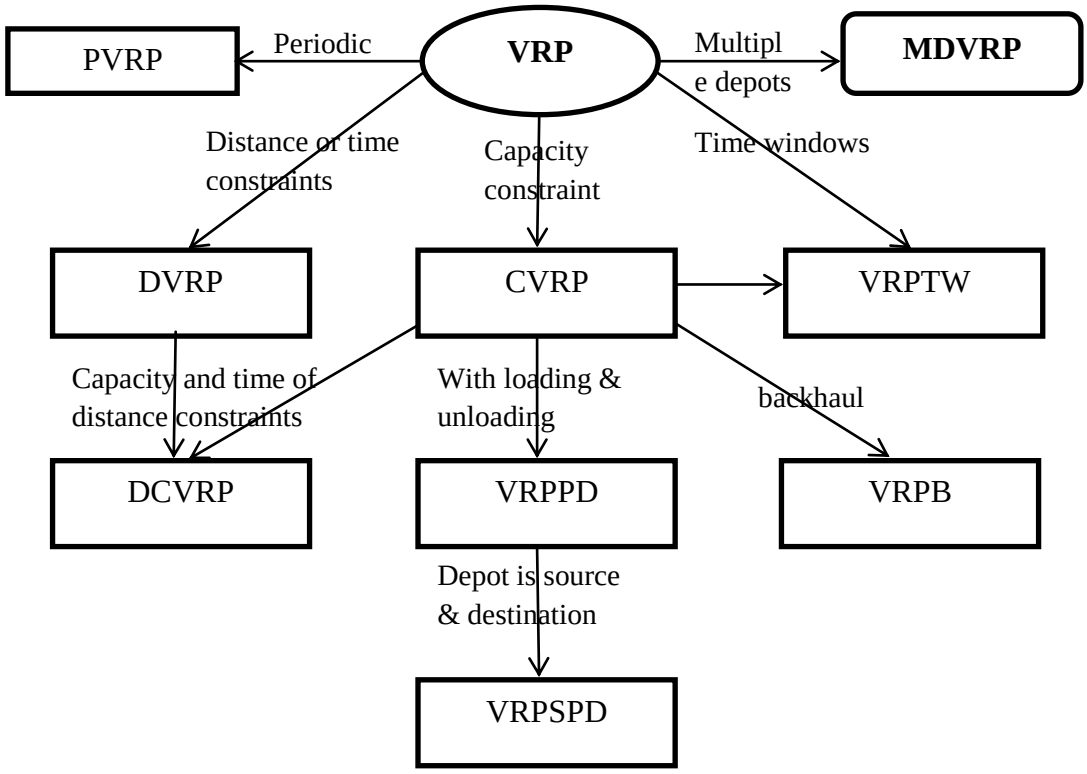

Figure 3. Different variants of the VRP

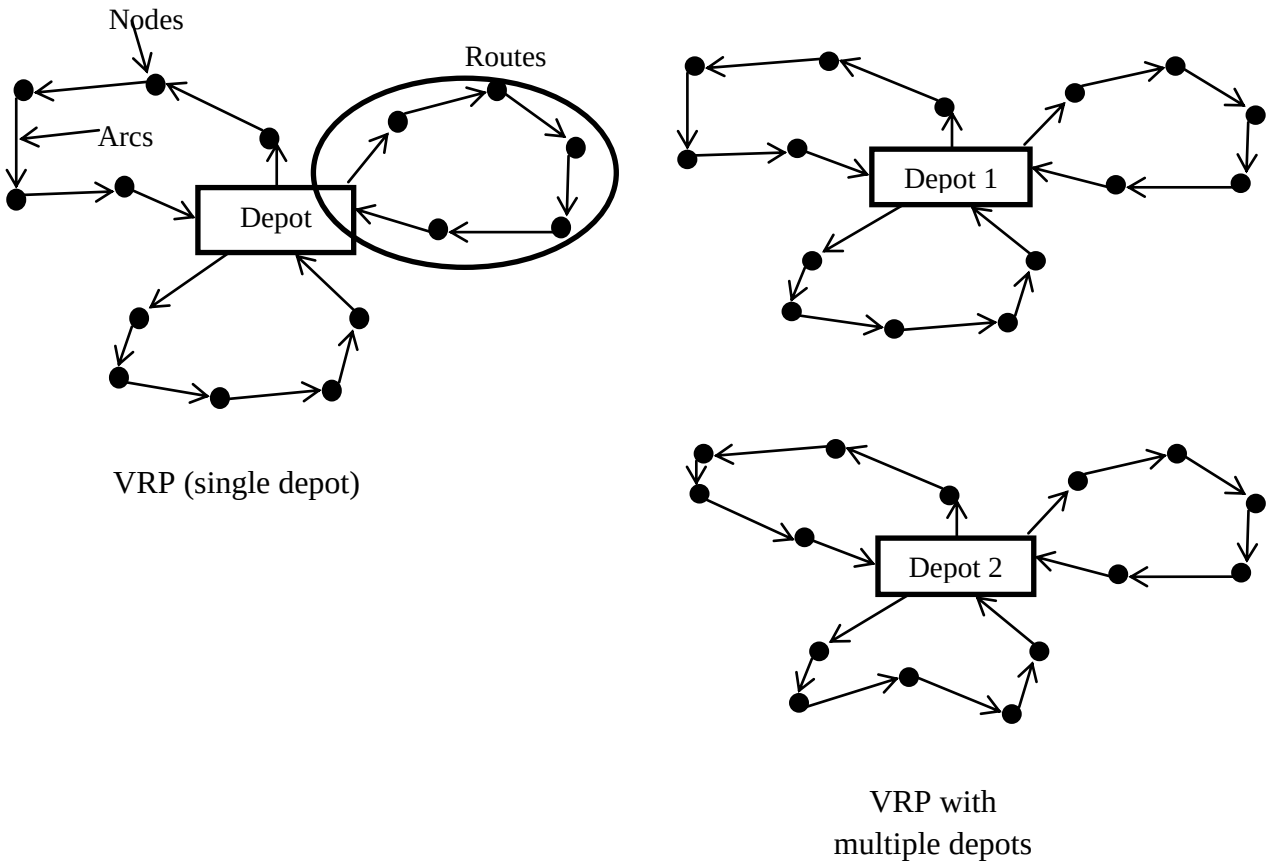

Figure 4. Comparison between VRP versus MDVRP

Next sections, will define the different versions of VRP and MDVRP, which have their own characteristics.

Capacitated Vehicle Routing Problem (CVRP)

CVRP is the elemental variant of Virtual Routing Problem, and can be defined as a complete graph G = (V, E), in which V = {0,...,n} is the vertex set and 'E' is the arc set. Vertices 1,...,n represent the clients, having a known and deterministic demand 'q_i', and V(0) corresponds to the depot. A travel cost associated to each arc belongs to 'E' between nodes i and j is denoted by C_{ij} (Toth & Vigo, 2001). CVRP takes into account usually a homogeneous fleet of 'm' identical vehicles with limited capacity. The CVRP calls out for the determination to find the minimum travel cost of a set of 'K' routes, when:

1. Each vehicle route starts and ends at the same depot.

2. Each client should be visited exactly once,
3. The sum of demands of clients should not exceed the vehicle capacity ‘Q’.

One universal assumption made for all the VRP variants (according to previous literature) is that every vehicle belongs to a particular depot and has to start and end its tour at that same depot.

Vehicle Routing Problem with Backhauls (VRPB)

In this VRPB, clients are divided into two classes: (i) Linehaul clients, requiring a delivery of the given quantity of products and (ii) Backhaul clients, for which given amount of products ought to be picked up. In the VRPB, there exists a precedence constraint between both groups of clients, i.e., on every route, the linehaul clients have to be served before the backhaul clients. A non-negative demand, ‘d_t’ (t, kind of demand), to be delivered or collected, is related to every client. VRPB determines minimal cost set for 'K’ circuits, such that:

1. Standard conditions of VRPs (given above) have to be followed.
2. The separate sum of the linehaul and backhaul clients demands should be less than the vehicle capability ‘Q’; and
3. For every circuit the backhaul clients served after the linehaul clients.

The form of VRP backhauling with time windows (VRPBTW) has been also proposed in the past studies.

Distance - Constrained Vehicle Routing Problem (DVRP)

The another variant we have is called DVRP, wherein maximum length is the principle constraint rather than capacity. In particular, a non-negative length, l_{ij} or l_e is now related to each arc and it's far essential that for each route, the total length of the arcs have to be much less than the maximum tour (route) length, L. Hence, the goal in the DVRP is to reduce the total route length. The case with both capacity and distance constraints is called as Distance-Constrained VRP (DCVRP).

VRP variant with Pickup and Delivery (VRPPD)

In VRPPD, different vehicles, located at multiple points, fulfil a set of clients requests. It can be outline through thinking about some variables i.e a point for pick up, a factor for delivery and a demand to be fulfilled among these points. In the basic VRPPD, a client 'i' has two properties 'd_i' and 'p_i' i.e. the required demand of the corresponding items to be delivered and to be picked up. Let 'O_i' is the point of origin for delivery and 'D_i' is the destination point for pickup. It is usually assumed, that the delivery must be performed before the pickup in order to follow the capacity constraint. Therefore, the load presently a vehicle has before arriving at next location can be calculated as:

[initial load- demands that are delivered + the demands already picked up]. The VRPPD includes locating minimal cost set as:

1. It follows the standard situations of VRPs (given above)
2. Clients 'O_i', when distinct from depot, must be served in the identical circuit before client'i';
3. The client 'D_i', when different from depot, must be served in the same circuit and after client 'i'.

Usually, beginning and the terminating points of the demands are common, and the case is known as the VRP with Simultaneous Pickup and Delivery (VRPSPD). The case of VRPPD with time windows is an another variant named as VRPPDTW.

Periodic Vehicle Routing Problem (PVRP)

PVRP is the case, where the simple VRP is generalized by way of extending the scheduling service period for N days. Thus, vehicles won't go back in the same day it departs. Additionally, every client can be visited more than once in this N-day period. The PVRP can be formulated as (Francis, Smilowitz, and Tzur, 2008): A Period set P = [δ1, . . . , δN] of N days that represents the planning period. A schedule is a set of days inside the planning period in which client services are fulfilled. Allocating a client to a schedule suggests that the client will acquire services in every day of that schedule. Also, each client has a fixed day to day demands that must be fulfilled by exactly one vehicle in only one visit. Hence, each client can be visited more than one time in a specific schedule. Hence to

limit the global cost; a feasible route of an ordered sequence of services must satisfy the subsequent constraints:

1. Standard VRP constraints.
2. The sequence of clients have to be visited by the same vehicle during the day 'δn'. Time to
3. travel the client sequence should not exceed the vehicles daily service time.

Vehicle Routing with Time Windows (VRPTW)

VRPTW is a vital version of VRP that has been attracted many researchers in the last 10 - 12 years. The VRPTW moreover has restrictions on the service time i.e. a time period referred to as time window is associated to each client. Therefore, the time instant for a route consists of: (i) the time wherein vehicle departs from depot, (ii) the total tour time, and (iii) the service time for each client (Toth & Vigo, 2001). The service for each client has to begin within the allotted time interval and for that time vehicle should stop at the client location. Moreover, vehicle will wait, if it reaches earlier than the service time starts. Also, the time window constraint induces an oblique orientation to each tour even if they're symmetric. Therefore, the VRPTW normally modelled as an asymmetric problem. VRPTW calls to discover a minimal cost set of 'K' simple circuits, such that:

1. It follows the same old constraints of VRPs (universal assumption).
2. Services for all clients should provoke in the time period (time window) and each vehicle have to halt for some time instant 's_i' at each client location.

Split Delivery Vehicle Routing Problem (SDVRP)

The Split Delivery VRP (SDVRP) was first proposed by Dror & Trudeaut (Dror, Laporte, and Trudeau, 1994), they advised a manner to generate savings allowing split deliveries. In this context, to serve the same client by different vehicles as long as the service plan reduces the total cost. This relaxation may be very important (Archetti & Speranza, 2008), in which the clients order sizes are as big as the capacity of the vehicle. A VRP can be remodeled right into an SDVRP through dividing each clients order into small orders. It gives a feasible solution, by satisfying all VRP constraints, besides a client may be served by more than one vehicle. The goal is to determine minimal of:

1. the overall cost of all routes,
2. the vehicle fleet
3. the travelling time required to serve all clients.

Stochastic Vehicle Routing Problem (SVRP)

The Stochastic VRP (SVRP) by means of E. Hadjiconstantinou and D. Roberts (E. Hadjiconstantinou & D. Roberts, 2001), differs from the VRP, due to one or more random variables within the system. Common examples of stochastic elements are (Gendreau, Laporte, and Eguin, 1995):

1. **Stochastic clients:** Sometimes, the valid clients sets aren't recognised. So, each client has a probability of being present or absent i.e. p_i and $1 - p_i$ respectively.
2. **Stochastic demands:** For each client, the demand 'd_i' is random.
3. **Stochastic travel times:** Service times 'δ_i' and travel times 't_{ij}' are random variables.

In SVRP, two approaches are made to find a solution:

(1) find a solution before knowing the realizations of the random variables (2) take recourse or corrective action when the values of the random variables are known. When the data is random, it isn't possible to satisfy all constraints for all realizations. So, the decision maker either satisfies some of the constraints with a given probability, or takes a few corrective actions on violation of a few constraints. For example; within the capacity constrained SVRP (with collections), vehicle might also return to the depot for possible corrective actions: (i) while the vehicle is full, and returns to unload the collection, and then resumes the service plan (ii) while the vehicle is full and returns to re-optimize the remaining part of the planned route (iii) vehicle is not full, but it is known that going to the next client could exceed its capacity. The purpose of the problem is to limit the vehicle count, the sum of routes taken and the sum of travel time required to service all clients with random values.

Multiple Depot Vehicle Routing Problem (MDVRP)

A company may also have several service points from where it serves its clients, therefore the problem is known as Multiple Depot VRP (MDVRP). Since there may be a large number of service points (depots) so it is hard to find the particular service point for clients service without violating the capacity

constraint. Hence grouping is done and clients are grouped (clustered) around depots according to the distance between clients and the depots. However, if the depots and the clients are intermingled then a Multi-Depot Problem can be solved.

The goal is to meet all clients' demands of products from several depots, whilst minimizing the vehicle count, the overall travel time and the travel distance. A feasible solution can be observed if each route follows the VRP constraints, also starts and stops at the same depot. The minimum-cost of the route is calculated like as standard VRP.

3. Solution Techniques

There are some solution techniques that are widely used to solve various variants of VRP and MDVRP.

3.1 Combinatorial Optimization Technique

3.2 Evolutionary approach

3.3 Multi-objective Optimization Technique

3.4 Heuristic/Meta-heuristic Techniques

3.5 Parallel Computation

3.1 Combinatorial Optimization Technique

These are the problems in which, a set of feasible solutions is discrete or it can be reduced to make discrete, so that a best optimal solution can be obtained that minimizes the given function. Branch and Bound or Branch and Cut are a few techniques to solve these problems, however not large and complicated problems. As the problem complexity increases, the algorithm’s cost drastically increases and optimality of the solution reduces, which makes the solution non-feasible. Other methods to optimize these problems include determining a suboptimal solution in an inexpensive time by using Heuristics and Meta-heuristics strategies.

3.2 Evolutionary Approaches

EA (Goldberg, 1989; Jong, 2006), are the part of Evolutionary Computations, based totally on the biological evolution and uses iterative process to determine the optimal solution. The evolutionary

algorithm copies the natural evolutionary principles to drive its search toward an optimal solution. EAs uses a population of solutions as opposed to a single solution per iteration and the end results are also the population of solution, i.e., the Pareto-optimal set. A solution seeker can select any solution among the solutions in the Pareto optimal set so obtained. EA's commonly solves the optimization problems efficiently. Homberger and Gehring (Homeberger & Gehring, 1999) defined GAs to resolve the VRPTW. Later a two-phase meta-heuristic is proposed by them, in which in first phase GA minimizes the vehicle count, and the another phase compute the minimum overall distance using tabu search. Researchers like Berger and Barkaoui (Berger & Barkaoui, 2003) developed wonderful populations evolved from GAs, pursuing distinct objectives/goal to reduce: (i) the overall distance and (ii) the violations of the time window constraints.

3.3 Multi-objective Optimization Technique

Multi-objective is the procedure of optimizing two or more conflicting goals simultaneously that is subject to certain constraints. It may be found in almost all practical conditions, where the solution received minimizes some goals and maximizes others goals at the same time. Single solution can not simultaneously limit and maximize each objective of a well shaped multi-objective problem. Hence solution should be such that optimizes each goal to a particular extent. Finding such solutions and quantifying them by means of evaluating to others is the primary goal when solving a multi-goal optimization problem. Two preferred approaches of multiple objective optimizations strategies are: 1.Scalar Methods (Preference-based Classical Approach) 2. Pareto optimal method (Ideal Approach)

Scalar approach additionally referred to as choice-based method (Tan, Chew, and Lee, 2006) solves the problem, via constructing an aggregate objective function which is aggregation of all the objectives. Here, weight of an goal is proportional to the desire vector assigned to that objective. This method converts the multi-objective problem to single-objective problem by scalarizing an objective vector to a single composite objective function. Hence, by means of optimizing such objective function, one specific trade-off solution is obtained. This technique of fixing multi-objective problems is simple but relatively subjective. It can be mixed with heuristics or meta-heuristics for any single goal, according to the past studies. For many multi-objective problems on VRP, Scalar method has been used with many

heuristics (Blum & Andrea, 2003); genetic algorithms (Ombuki, Ross, and Hanshar, 2006), and nearby search algorithms (Murata & Itai, 2007).

The second approach called Pareto Optimal technique, which determines a non-dominated set of solutions. While shifting from one Pareto solution to some other, there encounters a certain loss or advantage in one or more objectives so that certain advantage or loss can be recieved in other(s). Pareto optimal solution sets are usually implement on single solutions due of their practical applications in real-life problems. The size of Pareto sets is generally increased when the number of goals increases. In many multi-objective problems such as routing, the Pareto technique is extensively used (Doerner, Focke, and Gutjahr, 2007; Geiger, 2008) (Jozefowiez, Semet, and Talbi, 2004) (Tan, Chew, and Lee, 2006) to solve the VRPs. Note that, the relative desire vector (preference-based) is highly subjective, therefore not so simple and easy to compute, whereas, the Pareto optimal technique is more realistic and less subjective.

3.4 Heuristics and Meta-heuristics Techniques

The VRP problem has been significantly studied and discussed considering its introduction and a large number of heuristics and meta-heuristics techniques were proposed for fixing the VRPs various problems. This segment will describe the possible computational techniques recommended in literature to solve the VRP problems.

Heuristics Approaches

Mainly proposed between the Nineteen Sixties and Eighties, a huge range of heuristics is proposed for positive solutions to the VRP. These techniques are easy to perform and can find better solutions to large and complicated combinatorial problems with relatively less computational attempt; but, fail to determine the global optimum solution. The solution obtained for the problem won't be the best among all real solutions, or it is able to be simply approximate to the exact solution. But it's still valuable because finding it does not require a long time. There exists number of well-known heuristic strategies for fixing a variety of combinatorial problems, such as: Clarke and Wright Savings algorithm, Gillett and Miller Sweep algorithm, Fisher and Jaikumar algorithm, Variable Neighborhood Search (Braysy, 2003).

- **Clarke and Wright** (Clarke and Wright, 1964) Savings algorithm is the best-known heuristic. Starting with an initial solution s_1, wherein each client is served by a distinctive route, then searches for and merges 2 route extremities to find a feasible route. The sweep method through Gillett and Miller (Gillet & Miller, 1974), explores the clients circularly in a growing polar angle around the depot and each client is added to this order at the end of the current route. Renaud and Boctor (Renaud & Boctor, 2002) evolved the sweep algorithm for the fleet size and mixed VRPs.
- Other heuristic known as route-first cluster-second (Beasley, 1983) technique performs in two stages, first constructs a large circuit to visit all clients after which reduce this large tour into several routes from the depot. Fisher and Jaikumar's (Fisher & Jaikumar, 1981) cluster-first route-second method, works in two steps i.e. developing feasible clusters of all the clients and then final routes are obtained by applying TSP on each cluster. Variable Neighbourhood Search (VNS) is a search based heuristic (Mladenovic & Hansen, 1997). The VNS method has been effectively implemented to resolve variations of VRP (Braysy, 2003; Polacek, Hartl, Doerner, and Reimann, 2004).

Meta-heuristics Approaches

Meta-heuristics are the general algorithmic framework designed to resolve complex optimization problems (Osman & Laporte, 1996). Since, few decades meta-heuristics emerging as a successful alternatives to classical heuristics for fixing complex optimization problem. An extensive variety of meta-heuristic strategies have been introduced for fixing the VRPs and this class of algorithms have two principal classifications including: simulated annealing, tabu search (Glover & Laguna, 1998), local search (Aarts & Lenstra, 2003); lies in the class of Single Solution Approaches of meta-heuristic class. Another is Population-based Approaches, which includes evolutionary algorithms (Kenneth & Jong, 2006) genetic algorithm (GA), neural network (NN), and Swarm Intelligence techniques.

- **Simulated Annealing (SA)** (Laarhoven & Aarts, 1987) is a relaxation approach for optimization problems, which locates accurate approximation to global values (minimal or maximum) for the given function in the search space. Osman's implementation became a success algorithm, which give good solutions, however did not compete with the tabu search algorithms proposed at that

time. Tabu Search (TS) explores the solution space by moving at each iteration from a solution 's' to the best solution in a subset of its neighbourhood N(s). Tabu search techniques were proposed extensively since the past 2-3 decades. It is delivered by the Willard (Willard, 1989), but soon supplanted by many algorithms Osman (Osman, 1993) and Taillard (Taillard, 1993), that's one of the best-known tabu search algorithm to solve the CVRP. Local Search (LS) development heuristic (Aarts & Lenstra, 2003) starts with an initial solution, explores the neighbourhood to determine a better technique to update the existing solution, and repeats till no improving solutions can be located inside the neighbourhood.

- **Neural networks (NN)** are the computational models made from small units interconnected to each other through some related weights, like neurons inside the brain. Artificial neural network (ANN) is a mathematical model based on biological NNs. A self-organizing map (SOM) is a form of ANN which is trained using unsupervised learning to produce a low-dimensional (usually 2-D), training samples, known as map. The pioneer work of Hopfield (Hopfield & Tank, 1986) for fixing TSP offers a start to solve the combinatorial problems. Later, the elastic net (EN) model by Durbin and Willshaw (Durbin & Willshaw, 1987), and the SOM Kohonen model (Kohonen, 1988) have been proved more efficient for solving TSP than the Hopfield model. Ghaziri (Ghaziri & Osman, 2006) have solved VRP and VRP with backhauls using NN-SOM.
- **Swarm Intelligence (SI) Techniques** Nature Inspired Computing (NIC) is an emerging technology, inspired by the natures behaviour in numerous situations, to solve the complex problems. Swam Intelligence is a sub area of NIC and based on the study of cooperating behaviour of simple individuals (e.g., ants, bird flocking, and bees and so on.) in diverse decentralized systems. The population includes simple individuals, can normally resolve complex tasks by interacting locally among the individuals and with their environments. Therefore, SI is a promising stream to develop powerful solutions to optimization problems.

1) **Ant Colony Optimization (ACO)** (Dorigo, Caro, and Gambardella, 1999) is a meta-heuristic approach stimulated by an analogy, that ants use pheromone trails for shortest paths when foraging for food. A moving ant marking the path with a trail of some essence referred to as pheromone on

the ground. An individual ant pass randomly and when it detects a laid pheromone trail it follows that route and reinforces the trail with its very own pheromone, which attracts more ants. The method is hence characterised with a fine loop, wherein the probability of an ant chooses a path increases with the number of ants moved on that path. Bullnheimer et. al. (B. Bullnheimer, 1997) proposed the Ant System (AS, stimulated by foraging behaviour of ants) algorithm for solving the problems in VRP. Then they proposed an improved AS algorithm in 1999 and confirmed the competitiveness to solve the VRP in a reasonable time (Bullnheimer, Hartl, and Strauss, 1999). ACO for solving VRP is equal to the AS implemented for the VRP. Therefore, the implementation of ACO with a little improvement in the original algorithm follows three predominant steps: (i) Tour creation (ii) Route improvement techniques (iii) Pheromone updating. ACO has been efficiently applied to various version of VRPs as mentioned in literature, CVRP, PVRP, SDVRP, VRPPD (Montemanni, Gambardella, Rizzoli, and Donati, 2005; Kumar & Panneerselvam, 2012), VRPTW (Rizzoli, Montemanni, Lucibello, and Gambardella, 2007; Snadhaya & Katiyar, 2013). Researchers additionally mixed ACO with other successful algorithms to resolve for large-scale complex VRPs. Reimann et al. (Reimann, Doerner, and Hartl) have developed a successful method named D-Ants (Decomposition-Ants) construct on the Savings based Ant System (SbAS) and competitive to the best-known Tabu Search technique in terms of solution quality and processing time.

2) **Particle Swarm Optimization (PSO)** is a population-based method proposed by Eberhart and Kennedy (Kennedy & Eberhart, 1995) and simulated through the flocking behavior of birds and fish schooling. PSO optimizes the problem by iteratively improving a population of candidate solutions called particles, moving around the large search-area; adjusting its position and velocity according to the neighbouring particles. Each particle's movement is influenced by its local best known position (pbest) and guided through the global best (gbest) known position and velocity of its neighbors within the search-space. This way, while keep updating for better positions within the neighborhood the swarm moves toward the best optimal solutions. PSO has been successfully implemented in lots of studies and application areas and achieved better results in a quicker, less expensive way as compared with different strategies. Kachitvichyanukul (Ai & Kachitvichyanukul,

2009) presented a PSO algorithm for VRPPD and compared the overall performance of their approach with different current meta-heuristics on some benchmark problems. They additionally (Ai & Kachitvichyanukul, 2009) used a similar PSO for the CVRP. PSO has also been applied to other logistics problems (Ai & Kachitvichyanukul, 2009), (Ai & Kachitvichyanukul, 2008).

3) **Bees Algorithm** was introduced by Karaboga Dervis (Karaboga, 2005) and later (Karaboga & Akay, 2011) changed it as a brand new meta-heuristic approach. This algorithm is inspired from the intelligent behaviour of bees attempting to find meals (nectar). A bee's colony is made up of a ruler (queen) with numerous male drones and a huge variety of female labourer (non-reproductive). The process of the queen is to mate with the drones and to start new colonies. The employee bees assemble the honeycomb. The duty of the young bees is to give food to the queen and to the male drones, clean and shield the complete colony and to build up more food. The characterized interacting collective behaviour of specialised individuals, allotted simultaneous responsibilities carried out, and self-organization, leads to the colony as a prepared teamwork system. The Artificial Bees Colony (ABC) is an iterative algorithm, where each agent (bee) is assigned to a solution (food source) that is generated randomly; then in each iteration, each agent uses neighbourhood operator to find a new solution. The fitness function (nectar amount) of the newly determined solution is evaluated per iteration; if it is found maximum then replace the preceding solution through the more recent one. Gomez et.Al. (Gomez & Salhi, 2014) supplied a new ABC algorithm for fixing the CVRP. Y. Marinakis et.Al. (Marinakis, Mariniki, and Dounias, 2008) proposed honey bees mating algorithm for VRP named it as HBMOVRP, which correctly solves the VRPs.

4) **Intelligent Water Drops (IWD)** is introduced by Hamed Shah-Hosseini (Shah-Hosseini, 2007) as an algorithm, inspired from natural water drops that flow within the rivers and that how they divert to find a superior path to their destination. IWDs have essential properties:

    1. Velocity (positive initial velocity): Less soil more velocity gains in IWD and vice-versa.
    2. Soil (initially zero): Soil to an IWD is added from place to place.

In IWD algorithm, the artificial water drops change their surroundings to select best path with less soil on its links i.e. also for the next place to move paths should have less soil. The solutions are therefore incrementally formulated by means of the IWD algorithm. Shah-Hosseini tested a modified IWD algorithm on TSP and received close-to-optimal solutions. Rayapudi (Rayapudi, 2011) also applied IWD algorithm for fixing economic load dispatch problem.

3.5 Parallel Computation

In parallel computation, large and complex issues are divided into many small issues in order to reduce the computation time by way of solving them concurrently. Usually, an algorithm is written and carried out as a flow of serial instructions which are executed one by one on a computer system, which takes very long time. Here the parallel computing techniques are essential which permits multiple processes to run simultaneously, as problem is split into smaller ones and distributed to numerous processors. Parallel computer systems may be labeled as multi-core and multi-processor computer systems that can execute more than one process on a single system, while clusters, grids, and MPPs use exceptional computer systems for the same computation. Parallel computing programs are more difficult to write than sequential programs (Jozefowiez, Semet, and Talbi, 2002) because concurrent computation introduces new software bugs, like race condition which is most common. Also, synchronization and verbal exchange between subtasks is one among the most important obstacles for a good parallel program.

4. Literature Review

A large amount of literature survey has been done to apprehend the MDVRPs and the existing methods which have been applied to solve MDVRPs in a better and efficient manner. It is determined in previous studies that the exact methods such as Branch and Bound, Branch-Cut-and-Price, Set-Partitioning and Column Generation, have a size limit and failed to deal when the dimension and the complexity of the problem increases. Therefore, to overcome this problem the literature gives a wealth of heuristic and meta-heuristic techniques to solve the MDVRPs. This segment exhibits a literature

survey of papers considering the single objective case and multiple objectives case of the MDVRP. The findings are mentioned under:

Over the last thirty-five years several heuristics have been proposed for the Vehicle Routing Problem by Gilbert laporte and Frederic Semet (1998). This article reviews the main classical heuristic for this problem: constructive methods, two-phase methods, improvement heuristics. Several comparative computational results are reported. J. Christopher Beck, et.al. presented the graph transformations for the Vehicle routing and Job shop scheduling problems.

Jairo R. Montoya-Torres et.al.(Jairo R. Montoya-Torres, Julian Lopez Franco, Santiago Nieto Isaza, Heriberto Felizzola Jimenez, Nilson Herazo-Padilla, 2015) presents a state-of-the-art survey on the vehicle routing problem with multiple depots (MDVRP). their review considered papers published between 1988 and 2014, in which several variants of the model are studied: time windows, split delivery, heterogeneous fleet, periodic deliveries, and pickup and delivery. The review also classifies the approaches according to the single or multiple objectives that are optimized. Some lines for further research are presented as well.

Hamed Shah-Hosseini (2009), presented the idea among the latest nature-motivated swarm-based enhancement calculations, viz. the Intelligent Water Drops (IWD) technique. IWD techniques mirror a portion of the procedures that occur in nature between the water drops of a stream and the dirt of the waterway bed. The IWD technique was initially presented in (Hamed Shah-Hosseini, 2007) in which the IWDs are utilized to tackle the Traveling Salesman Problem (TSP). The IWD technique has additionally been effectively connected to the Multidimensional Knapsack Problem (MKP), n-ruler confound, and Robot Path Planning (Duan et al., 2008).

The works of Kulkarni and Bhave (1985), and Carpaneto, Dell'amico, Fischetti, and Toth (1989) can additionally be considered as a feature of the pioneer works on exact methods for the MDVRP. The mathematical formulation proposed by Kulkarni and Bhave (1985) was later modified by Laporte. All the more as of late, Baldacci and Mingozzi (2009) proposed mathematical formulations for comprehending a few classes of vehicle routing problems including the MDVRP.

Dondo, Mendez, and Cerda (2003) proposed a mixed-integer linear programming (MILP) model to limit routing cost in the HFMDVRP, in which heterogeneous fleet of vehicles are accessible. The work of Kek, Cheu, and Meng (2008) proposes a mixed-integer linear programming model and a branch-and-bound methodology for the MDVRP with fixed fleet and pickup and delivery. The objective function is the minimization of the aggregate cost of routes. Cornillier, Boctor, and Renaud (2012) introduced a MILP model for the issue in which heterogeneous fleet of vehicles is accessible and with expansion of aggregate net revenue as objective function, while most extreme and least demands constraints are given.

Seth, Klabjan, and Ferreira (2013) concentrated an application case from the printed circuit board creation process displayed as a MDVRP with mobile depots. They displayed an exact algorithm in view of network-flow formulation and proposed an iterated tour partitioning heuristic. Branch-and-cut calculations were proposed by Benavent and Martínez (2013) and Braekers, Caris, and, Jenssens (2014). Previous authors concentrated additionally on considering the polyhedral structure of the problem which permitted the expansion of their technique to the location-routing problem (LRP), while last authors considered a dial-a-ride problem with various depots. The LRP with various depots was additionally considered by Contardo, Cordeau, and Gendron (2014) who proposed a cut-and-column generation technique for the capacitated case. Other arrangements of the MDVRP have been concentrated through exact algorithm. For example, Contardo and Martinelli (2014) contemplated the capacitated MDVRP with route length constraints.

Since the NP-hardness of the MDVRP, a few heuristic algorithms have been proposed in the literature. The principal works were distributed in the 1990's, so as to understand the capacitated version. Min, Current, and Schilling (1992) concentrated the form of the MDVRP with backhauling and proposed a heuristic technique based on problem decomposition.

The HFMDVRP, in which heterogeneous fleet of vehicles is accessible have caught the consideration of scientists since the work introduced by Salhi and Sari (1997). Irnich (2000) proposed a set covering heuristic coupled with column generation and branch-and-price algorithm for cost minimization for the heterogeneous fleet and pickup and delivery MDVRP. Dondo and Cerda (2007) proposed a MILP

model and in addition a three-stage heuristic. Some time recently, a preprocessing stage for node clustering is performed and a more smaller cluster-based MILP issue detailing is produced.

The work of Gulczynski, Golden, and Wasil (2010) built up an integer programming-based heuristic. The target of this review was to determine the reduction in traveled distance that can be accomplished by permitting split deliveries among vehicles based at a similar depot and vehicles based at various depots.

The multi-depot capacitated vehicle routing problem with split delivery (MDCVRPSD) is contemplated by Liu, Jiang, Fung, Chen, and Liu (2010). A numerical model is proposed in view of a graph model. The target function is the minimization of movements of empty vehicle. A greedy algorithm is proposed as all things considered, with a specific end goal to comprehend large-scale instances.

The primary meta-heuristic was proposed in the work of Renaud, Laporte et al. (1996) who concentrate the MDVRP with the imperatives of vehicle limits and maximum duration of routes (e.g. the time of a route can not surpass the maximum working time of the vehicle). The target to be advanced is the total operational cost.

Recently, Yücenur and Demirel (2011a) proposed geometric shape based genetic clustering algorithm for the traditional MDVRP. The system is compared with the nearest neighbor algorithm. Their examinations demonstrated that their calculation gives a superior clustering performance as far as the distance of each client to every depot in clusters, in a significantly less computational time.

In the overview by Gendreau, Potvin, Bräumlaysy, Hasle, and Løkketangen (2008) concentrated on the use of meta-heuristics for tackling different variations of the VRP, a short modification of the multi-depot issue is introduced. The proportionality between the MDVRP and the PVRP is additionally broke down.

A genuine transportation issue comprising on moving vacant on the other hand loaded compartments is contemplated by Tan, Chew, and Lee (2006). They called the issue as the truck and trailer vehicle routing problem (TTVRP), however in truth it compares to a variation of the MDVRP: the answer for the TTVRP comprises of finding a total routing schedule for serving the jobs with least routing distance

and number of trucks, subject to time windows and accessibility of trailers. These creators understood the multi-objective case utilizing a hybrid multi-objective evolutionary algorithm (HMOEA) with particular genetic operators, variable-length representations and local search heuristic.

Lau, Tang, Ho, and Chan (2009) considered the multi objective issue in which the travel time is not a constraint but an objective function to be improved in the model together with the total traveled distance. The proposed solution technique is a hybrid meta-heuristic named fuzzy logic guided non-dominated sorting genetic algorithm (FL-NSGA2). The system utilizes fuzzy logic to powerfully conform the probabilities of mutation and crossover. Calculation is compared with the understood calculations non-dominated sorting genetic algorithms 2 (NSGA2), strength Pareto evolutionary algorithm 2 (SPEA2) with and without fuzzy logic and the micro-genetic algorithm (MICROGA) with and without fuzzy logic. Tests demonstrated that the proposed FL-NSGA2 method beat the other methods.

In their work, a few search techniques, branch-and-bound, standard GA (i.e., without the guide of fuzzy logic), simulated annealing, and tabu search technique are embraced to compare with FLGA in arbitrarily produced data sets. After, effects of their examinations demonstrate that FLGA beats alternate strategies. The main creators considered the goals of limiting the total cost and the number of vehicles, while the last creators considered the total distance, the idle capacity of vehicles and the quantity of externalized deliveries.

Mehdi Adelzadeh, Vahid Mahdavi Asl, and Mehdi Koosha (2014) introduced a mathematical model and a new heuristic solution method for solving multi-depot vehicle routing problem with time windows and different types of vehicles. In this problem, depots must serve clients between their fuzzy time windows with vehicles having different capacities, velocities, and costs. And, Ran Liu, Zhibin Jiang, and Na Geng (2014) presented a new hybrid genetic algorithm for finding the routes that minimize the traveling cost of the vehicles.

Mohammad Mirabi (2015) provides one definition of periodic vehicle routing problem for single and multi depot conforming to a wide range of real-world problems and also develops a novel hybrid genetic algorithm to solve it. The proposed algorithm applies a modified approach to

generate a population of initial chromosomes and also uses an improved heuristic called the iterated swap procedure to improve the initial solutions. P. Stodola, and J. Mazal (2015), introduced the meta heuristic solution to the MDVRP using the Ant Colony Optimization (ACO) algorithm. The initial section of the paper presents the original algorithm. It introduces pivotal principles of the algorithm, along with experiments and achieved results on benchmark instances.

The work of Eduardo Lalla-Ruiz, Christopher Exposito-Izquierdo, Shervin Taheripour, and Stefan Vob (2016) proposed a new mixed integer programming formulation for the multi depot open vehicle routing problem by improving some constraints from the literature and proposing new ones. Jian Li, Yang Li, and Panos M. Pardalos (2016) formulated an integer programming model with the minimum total traveling cost under the constraints of time window, capacity, and route duration of the vehicle, the fleet size and the number of parking spaces of each depot. Due to NP–Hardness of problem, a hybrid genetic algorithm with adaptive local search is proposed to solve it. Also, Jiahong Zhao, and Fumin Zhu (2016) proposed a multi depot vehicle–routing model with the minimization of total cost and total risk. This model is formulated through the two-commodity flow formulation, and it is characterized by simultaneously planning tours and vehicle acquisitions for the explosive waste collection, and designing the return-trips between collection centers and recycling centers. To solve this bi-objective problem, a modified lexicographic weighted Tchebycheff method is proposed.

The primary objective of Vikas Agrawal, Constance Lightner, Carin Lightner-Laws, and Neal Wagner (2017) works is to explore solutions for real-world VRPs with a heterogeneous fleet of vehicles, multi depot subcontractors (drivers), and pickup/delivery time window and location constraints. They use a nested bi-criteria genetic algorithm to minimize the total time to complete all jobs with the fewest number of route drivers. Their model will explore the issue of weighting the objectives (total time vs. number of drivers) and provide Pareto front solutions that can be used to make decisions on a case-by-case basis.

In the work of Eneko Osaba, Xin-She Yang, Fernando Diaz, Enrique Onieva, Antonio D. Masegosa, and Asier Perallos (2017), a real-world newspaper distribution problem with recycling

policy is handled. To meet all the complex restrictions contained in such a problem, it has been modeled as a rich VRP, which can be more specifically considered as an asymmetric and clustered VRP with simultaneous pickup and deliveries, variable costs, and forbidden paths. Eduardo Uchoa, Diego Pecin, Artur Pessoa, Marcus Poggi, Anand Subramanian, and Thibaut Vidal (2017) proposed a new set of instances ranging from 100 to 1000 clients, designed in order to provide a more comprehensive and balanced experimental setting because the recent research on the CVRP is being slowed down by the lack of a good set of benchmark instances.

Silvia Schwarze, and Stefan VoB (2013) introduce that as no resource restrictions are imposed, the skill VRP tends to produce TSP-like solutions. To determine better balanced solutions, they propose two new techniques. First they propose a minmax model that aims at minimizing the maximal vehicle tour length. Second they introduce a two-step process combining the minmax technique with a distance constrained model. Finally, their experiments describes that these techniques lead to improvements in load balancing and resource utilization. In the work of Wei Zhou, Tingxin Song, Fei He, and Xi Liu (2013), they propose a genetic algorithm to determine the biobjective VRP with time windows simultaneously considering total distance and distance balance of active vehicle fleet. With the help of tournament selection, one-point crossover, and migrating mutation operator, the problem is solved.

5. Motivation

The MDVRP is tougher and sophisticated than the single-depot VRP. The variant with multiple depots appears first in the literature on the works of Kulkarni and Bhave (1985), and Carpaneto, Dell'amico, Fischetti, and Toth (1989). Since then, large amount of studies has been published within the shape of journal paper, conference paper, research/technical report, thesis or book. To the best of their knowledge, despite the large amount of research papers published, there is no rigorous literature survey found exclusively devoted to the solutions to MDVRP with swarm intelligence techniques. A quick overview of academic work was proposed by many researchers, presenting the most representative research papers. Most of the work has been focused on the minimization of cost, distance or time. Besides, these authors concentrate on the problem definition, solution methods (dividing them into

exact algorithms, heuristics and meta-heuristics) and point out a few problem variants. In reality, all previous research work on MDVRP was limited to theoretical study but now it has become very relevant to our real life problems. To overcome these problems, researchers proposed Swarm Intelligence (SI) based techniques to solve MDVRP, which gives near optimal solutions to the problem in comparison to other existing methods. Since then SI techniques emerged as successful approaches to solve large and complex optimization problems and also became an important research area to work on. Therefore, this study is motivated by both MDVRP's practical importance with considerable difficulty and the advantageous features of SI techniques to solve the complex optimization problems.

MDVRP solutions can also be applied to routing where n number of sending/receiving nodes are being served by m routers (m<<n). This motivates us to explore the possibility of implementation of the solutions to MDVRP to solve network routing problem of an organization, such as Dayalbagh Educational Institute (DEI). An organization would have a number of routers which would all be interconnected, and which would serve a big number of nodes. This internetwork of routers could be connected to outer internet via a gateway. In such a scenario, route balancing of routes serving certain nodes would be required. It would be seen as to how our solutions to MDVRPs fares to solve such a problem. A particular case study of DEI's network would be taken to carry out this experiment.

As can be seen, there is a clearly growing trend displaying the growing interest in this field. It is reasonable to expect that in the coming years the MDVRP will acquire an ever large amount of attention.

6. Objectives

The prime objective is to obtain the solutions and to optimize the Multiple Depot Vehicle Routing Problem having multiple objectives such as obtaining the shortest path, least cost path, minimize the time spent in serving all clients, minimize the number of vehicles, load balancing etc., by using various swarm intelligence techniques and compare the results obtained. It is proposed to:

- Study how MDVRP can be solved and optimized using various swarm intelligence techniques such as Ant Colony Optimization, Intelligent Water Drops etc. and obtain serial solutions to it.
- Explore the possibilities of having parallel solutions for our defined problem.
- Implementation of the proposed techniques and algorithms on the variants of MDVRPs.
- Compare existing strategies with the proposed tehniques on standard benchmark problems of MDVRP such as P06, P09, PR03, PR08 etc.
- **Explore the possibilities of implementation of the solutions of MDVRP on route balancing of routers serving various sending/receiving nodes.**

7. Platform

MATLAB will be used as the main tool to run simulations, and to compute and analyse the results of the newly developed algorithms. Local Area Multicomputer (LAM)/Message Passing Interface (MPI) is being proposed to be used for overall simulations of serial and parallel implementations of the algorithm.

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